Seeing with Sound: Asteroseismology

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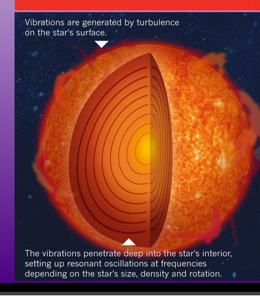

ABSTRACT

By introducing a new and improved photometric mode identification formula for pulsating stars, we describe the effect of pulsation in the light output of a pulsating star. The calculation we did shows the dependence of the variation in the observed luminosity on the surface area, surface normal and variation in temperature caused by nonradial pulsation.

INTRODUCTION: Asteroseismology is a study of sesmic waves in stars with the aim of using these to infer the interior physics of stars. Photometric mode identification is very important in asteroseismology because the number of modes identified determines the amount of information we get from a particular star. Watson(1987,1988) developed analytic expression of observed flux for non radially pulsating stars. In his formalism, Watson showed the relative importance of local temperature, geometry, pressure and limbdarkening in the predicted surface flux changes. But in his calculation, he considered at a particular layer $\tau = 2/3$. Based on Watson (1987, 1988) and Medupe et al. (2009) we developed an improved mode identification formula and we show the effect of all the layers of the atmosphere in the light output of the pulsating star, where the top layer contributes the most.

Basic Assumptions

Spherically Symmetric and hydrostatic equilibrium, Local thermodynamical equilibrium (LTE), linear non-adiabatic and plane parallel atmosphere in a star. When a star pulsates:

- Surface Normal, Surface area and Temperature all vary.

Flux Variation

The surface flux coming out of a non-pulsating star is:

$$F_\lambda = \int\int \mu I_\lambda(\mu,\phi) d\Omega\ , \tag{1}$$

where $\mu = \cos\theta$ is the directional cosine, I_λ is light intensity. A pulsating star has perturbation in flux given by:

$$\delta F_\lambda = \int\int \mu_0 I_\lambda d\delta\Omega + \int\int \mu_0 \delta I_\lambda d\Omega_0\ , \tag{2}$$

where δ is linear perturbation.

$$\frac{\delta F}{F} = \frac{1}{F}\int\int \mu\delta I_\lambda d\Omega + \frac{1}{F}\int\int \mu I_\lambda d\delta\Omega\ . \tag{3}$$

The New Photometric Mode Identification Formula

The variation in luminosity can be written as:

$$\frac{\delta L_\lambda}{L_\lambda(0)} = \frac{\delta F_\lambda}{F_\lambda(0)} + \frac{\delta A_\lambda}{A_\lambda}, \tag{4}$$

where $\frac{\delta A_\lambda}{A_\lambda}$ is the variation in the surface area. Then,

$$\begin{aligned}\frac{\delta L_\lambda}{L_\lambda(0)} = {} & \frac{Y_l^m(\theta_0,\phi_0)}{2H_{\lambda 0}(\lambda)}\int_0^1\int_0^\infty \delta\tilde{B}_\lambda(r) p_l(\mu) e^{\frac{-\tau_\lambda}{\mu}} d\tau_\lambda d\mu \\ & - \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty \frac{\delta\tilde{\kappa}_\lambda}{\kappa_\lambda} p_l(\mu)(I_\lambda - B_\lambda) e^{\frac{-\tau_\lambda}{\mu}} d\tau_\lambda d\mu \\ & - \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty \frac{(1-\mu^2)}{\mu}\frac{dp_l(\mu)}{d\mu}\frac{\delta\tilde{r}}{r}(I_\lambda - B_\lambda) e^{\frac{-\tau_\lambda}{\mu}} d\mu d\tau_\lambda \\ & - \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty 2\mu p_l(\mu) B_\lambda(\tau)\frac{\delta\tilde{r}}{r} e^{\frac{-\tau_\lambda}{\mu}} d\mu d\tau_\lambda \\ & + \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty (1-\mu^2) B_\lambda(\tau)\frac{dp_l(\mu)}{d\tau}\frac{\delta\tilde{r}}{r} e^{\frac{-\tau_\lambda}{\mu}} d\mu d\tau_\lambda .\end{aligned} \tag{5}$$

where $p_l(\mu)$ is the Legendre polynomial, Y_l^m is the spherical harmonic function, B_λ is the Planck's function, l is the degree of the mode and specifies the number of surface nodes and m is the azimuthal order of the mode that shows the number of nodal lines crossing the symmetry axis of pulsation.

Results

Simplifying eqn. (5), the observed variation in luminosity for pulsating stars becomes:

$$\begin{aligned}\frac{\delta L}{L_\lambda(0)} = {} & \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty \delta\tilde{B}_\lambda(r) P_l(\mu)\Big(1-\frac{\tau_\lambda}{\mu}\Big) d\tau_\lambda d\mu \\ & - \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\int_0^1\int_0^\infty \frac{\delta\tilde{\kappa}_\lambda}{\kappa_\lambda} P_l(\mu)\Big(1-\frac{\tau_\lambda}{\mu}\Big)\frac{dI_\lambda}{d\tau_\lambda} d\tau_\lambda \mu d\mu \\ & - \frac{Y_l^m(\theta_0,\phi_0)}{2H_\lambda(0)}\Big\{\int_0^1 \Big(\frac{1-\mu^2}{\mu}\Big)\frac{dP_l}{d\mu} f(\mu)\mu d\mu \\ & + \int_0^1 \Big[2P_l(\mu)\mu - (1-\mu^2)\frac{dP_l}{d\mu}\Big] g(\mu) d\mu\Big\}.\end{aligned} \tag{6}$$

where,

$$f(\mu) = \int_0^\infty \frac{\delta\tilde{r}}{r}\frac{dI_\lambda}{d\tau_\lambda}\Big(1-\frac{\tau_\lambda}{\mu}\Big)\frac{d\tau_\lambda}{2H_\lambda(0)}, \tag{7}$$

and

$$g(\mu) = \int_0^\infty \frac{\delta\tilde{r}}{r}\frac{B_\lambda}{2H_\lambda(0)}\Big(1-\frac{\tau_\lambda}{\mu}\Big) d\tau_\lambda, \tag{8}$$

For radially oscillating stars where $l = 0$, the geometric term (the term in the square bracket) in the above equation, the variation in the surface normal vanishes. The final equation for observed luminosity is given above where it shows all the variations during non radial pulsation including the variation in the surface normal, where this term vanishes for radial pulsation, variation in the surface area and temperature

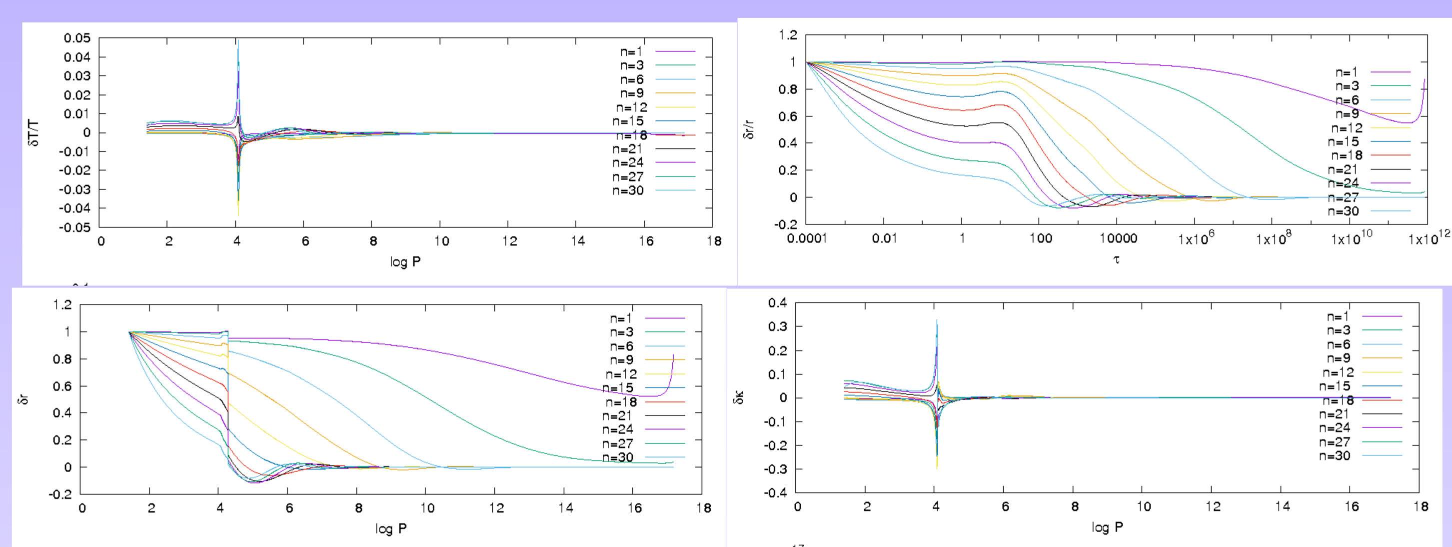

FIGURE 1: Top left panel: Figure showing how the temperature eigen function behaves in a star for an equilibrium model with $T_{\rm eff} = 8340$K and $logg = 4.3185$. Bottom left panel: The displacement eigen function as a function of pressure in the atmosphere of a pulsating star. Top right panel:Figure showing the behaviour of the displacement eigen function as a function of optical depth τ in the atmosphere of the star. Bottom right panel: The variation in the eigen function of opacity as a function of depth ($logP$). Where the n values are related to frequency of pulsation.

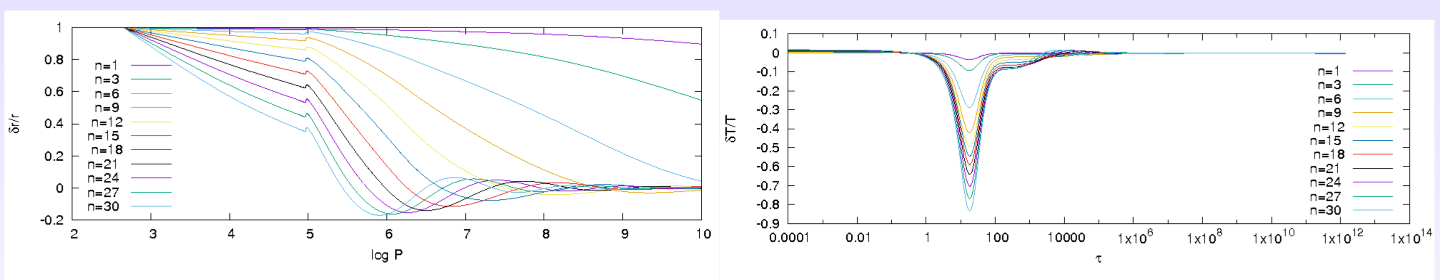

FIGURE 2: Left panel: Figure showing how the displacement eigen behaves in a star for an equilibrium model with $T_{\rm eff} = 6430$K and $logg = 4.35$. Right panel: The temperature eigen function as a function of optical depth τ in the atmosphere of a pulsating star.

Conclusion

When a pulsating star is observed, there is a manifestation of changes in brightness. The observed brightness changes are caused due to changes from geometric effects (distortion in the surface area, for non radial pulsation, variation in the surface normal). Other changes that causes variation in brightness are thermodynamical changes which are associated to optical depth perturbations and $(I_\nu - B_\nu)$. Hence, our calculations are developed by taking in to account the changes mentioned.

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